

GAUGE DEFECT NETWORKS IN TWO-DIMENSIONAL CFT[†]

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ABSTRACT. An interpretation of the gauge anomaly of the two-dimensional multi-phase σ -model is presented in terms of an obstruction to the existence of a topological defect network implementing a local trivialisation of the gauged σ -model.

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1. INTRODUCTION

The Symmetry Principle and the Gauge Principle figure among the most fundamental ideas of local QFT. In particular, they serve to distinguish theories with a non-anomalous realisation of classical symmetries, admitting descent to the space of orbits of an action of the symmetry group on the space of states. This note gives an account of recent progress in realising the Principles in 2d CFT in the mixed geometro-cohomological framework developed by Gawędzki *et al.* and adapted in Refs. [RS09, Sus11] to the study of CFT defects. As we shall argue, it is through application of the ensuing correspondence between defects and CFT dualities along the lines of Ref. [Sus12] that the categorical construction of Refs. [GSW11, GSW10, GSW12] of the gauged CFT is justified and brought to completion. Our discussion emphasises the naturalness of Gawędzki's approach and paves the way towards a rigorous description of T-duality and T-folds.

2. THE MULTI-PHASE σ -MODEL

The mono-phase 2d non-linear σ -model is a theory of maps $X \in C^1(\Sigma, M)$ from a compact closed 2d metric manifold (Σ, η) to a metric manifold (M, g) supporting

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a bundle gerbe \mathcal{G} . The gerbe, as defined in Refs. [Mur96, MS00], is a geometric realisation of an integral class

$$[\frac{1}{2\pi} \mathbf{H}] \in H^3(M).$$

It gives rise to a Cheeger–Simons differential character $\text{Hol}_{\mathcal{G}}(X)$ defined as the image of $[X^*\mathcal{G}]$ under the isomorphism $\tilde{H}^2(\Sigma, \text{U}(1)) \xrightarrow{\cong} \text{U}(1)$. The theory is determined by the Principle of Least Action applied to the functional

$$S_{\sigma}[X; \eta] := -\frac{1}{2} \int_{\Sigma} g(dX \wedge \star_{\eta} dX) - i \log \text{Hol}_{\mathcal{G}}(X)$$

in which the Hodge operator \star_{η} acts on the first and the metric g contracts the second tensor factor in $dX(\sigma) = \partial_a X^{\mu}(\sigma) d\sigma^a \otimes \partial_{\mu}|_{X(\sigma)}$ (in local coordinates $\{\sigma^a\}^{a \in \{1,2\}}$ on Σ and $\{X^{\mu}\}^{\mu \in \{1, \dim M\}}$ on M). The class $[\frac{1}{2\pi} \mathbf{H}]$, necessary for the cancellation of the Weyl anomaly, is represented by a 2-cocycle in the Deligne hypercohomology of M , and it is in this guise that gerbes first entered the description of the σ -model in Refs. [Alv85, Gaw88], leading to a classification of field theories with fixed (M, g, \mathbf{H}) by elements of $H^2(M, \text{U}(1))$, and to a geometric quantisation of the σ -model through transgression in Ref. [Gaw88].

Drawing inspiration from models of spin lattices with lines of spin frustration and from orbifold string theory, we are led to consider, after Ref. [FSW08], mappings from Σ to a disjoint union of manifolds M , with discontinuities along lines $\ell \subset \Sigma$ at which the component of the energy-momentum current generating diffeomorphisms of Σ that preserve ℓ is continuous, whence the name conformal defects. Amidst these, we find topological defects with the property that all components of the current are continuous at ℓ . A formal path-integral argument of Ref. [RS09] shows that in a factorisable CFT with defect lines we are bound to consider intersections of the latter. Together with defect lines, they compose an oriented graph $\Gamma \subset \Sigma$ that splits Σ into patches. Thus, we arrive at the definition of the multi-phase σ -model of Ref. [RS09]: It is a theory of C^1 -smooth maps X that send the patches to a metric space (M, g) , defect lines to a smooth space Q with smooth maps $\iota_1, \iota_2 : Q \rightarrow M$ and $\omega \in \Omega^2(Q)$, and defect junctions of valence $n \geq 3$ to a smooth space T_n with smooth maps $\pi_n^{k, k+1} : T_n \rightarrow Q$, $k \in \mathbb{Z}/n\mathbb{Z}$ subject to additional consistency constraints. The target space

$$\mathcal{F} := M \sqcup Q \sqcup T, \quad T := \bigsqcup_{n \geq 3} T_n$$

is the base of a geometric structure \mathfrak{B} from the 2-category $\mathfrak{BGrb}^{\nabla}(\mathcal{F})$, introduced and elaborated in Refs. [Ste00, Wal07], of abelian bundle gerbes with connection over \mathcal{F} . The structure consists of a 0-cell (gerbe) \mathcal{G} , a 1-cell (1-isomorphism)

$$\Phi : \iota_1^* \mathcal{G} \xrightarrow{\cong} \iota_2^* \mathcal{G} \otimes I_{\omega}$$

(for I_{ω} the trivial gerbe of curving ω), and 2-cells (2-isomorphisms)

$$\varphi_n : \circ_{k \in \mathbb{Z}/n\mathbb{Z}} \pi_n^{k, k+1} * \left(\Phi^{\varepsilon_n^{k, k+1}} \otimes \text{Id} \right) \xrightarrow{\cong} \text{Id}$$

(with $\varepsilon_n^{k, k+1} = \pm 1$ for an incoming/outgoing defect line with index k). The triple $(\mathcal{G}, \Phi, \varphi_n)$ is a geometric realisation of the 2π -integral class of the relative 3-form

$$\Theta := \mathbf{H} \oplus \omega \oplus 0$$

in the cohomology of the complex of cochain groups

$$C_{\text{dR}}^p(M, Q, T_n | \Delta_Q, \Delta_{T_n}) := \Omega^p(M) \oplus \Omega^{p-1}(Q) \oplus \Omega^{p-2}(T_n), \quad p \in \mathbb{N}$$

(here, $\Omega^{-1}(X) := \mathbb{R}^{\pi_0(X)}$ and $\Omega^{m<-1}(X) \equiv 0$) endowed with the differential

$$\widetilde{d}(\omega_M \oplus \omega_Q \oplus \omega_{T_n}) := d\omega_M \oplus (-d\omega_Q - \Delta_Q \omega_M) \oplus (d\omega_{T_n} - \Delta_{T_n} \omega_Q),$$

written for $\Delta_Q := \iota_2^* - \iota_1^*$ and $\Delta_{T_n} := \sum_{k \in \mathbb{Z}/n\mathbb{Z}} \varepsilon_n^{k, k+1} \pi_n^{k, k+1*}$, cf Ref. [Sus12]. The said theory is determined by the Principle of Least Action applied to the functional, given in Ref. [RS09],

$$S_\sigma[(X|\Gamma); \gamma] := -\frac{1}{2} \int_{\Sigma} g(dX^\wedge \star_\gamma dX) - i \log \text{Hol}_{(\mathcal{G}, \Phi, \varphi_n | n \in \mathbb{N}_{\geq 3})}(X|\Gamma),$$

in which the last term is a differential character expressed in terms of Deligne data of the $(\mathcal{G}, \Phi, \varphi_n)$. The definition enables us to label conformal defects with fixed $(M, g, \mathcal{G}, Q, \iota_\alpha, \omega)$ by classes in $H^1(Q, \text{U}(1))$, and (inequivalent) choices of the φ_n by elements of $\text{U}(1)^{\pi_0(T_n)}$. It also determines a geometric quantisation of (the twisted sector of) the σ -model through a relative variant of transgression described in Refs. [GR02, GR03, Gaw05, Sus11].

3. DEFECTS VS. DUALITIES

A relation between topological defects and CFT dualities has been predicted within the categorical approach to the quantisation of the CFT developed by Fröhlich, Fuchs, Schweigert, Runkel *et al.* in Refs. [FFRS04, FFRS07] and subsequent works. Its realisation in the geometric framework outlined above was worked out in the author's recent paper [Sus11], preceded by a case study in Ref. [RS09] of a class of defects in the Wess–Zumino–Witten–Gawędzki σ -model of Refs. [Gaw88, GR02, GR03, Gaw05]. It stems from the simple world-sheet intuition: A (space-like) circular defect ℓ_\circ separating two patches of the σ -model establishes a correspondence between states from the respective phases, represented by the Cauchy data localised at the circular components of the boundary of the patches joining at ℓ_\circ . A canonical analysis of the relation between the two sets of Cauchy data that follows from the gluing condition imposed at ℓ_\circ , as part of the variational principle, upon classical (*i.e.*, minimal) field configurations X demonstrates that the subspace

$$\mathfrak{J}_\sigma \subset \mathbb{P}_\sigma^{\times 2}$$

composed of pairs of states in cross-defect correspondence is isotropic with respect to $\text{pr}_1^* \Omega_\sigma - \text{pr}_2^* \Omega_\sigma$ (here, the $\text{pr}_\alpha : \mathbb{P}_\sigma^{\times 2} \rightarrow \mathbb{P}_\sigma$ are the canonical projections). The defect data induce a full-fledged (pre)quantum CFT duality, that is \mathfrak{J}_σ is a graph of a symplectomorphism preserving the Hamiltonian density and lifting to an isomorphism

$$\text{pr}_1^* \mathcal{L}_\sigma|_{\mathfrak{J}_\sigma} \cong \text{pr}_2^* \mathcal{L}_\sigma|_{\mathfrak{J}_\sigma}$$

between pullbacks of the prequantum bundle \mathcal{L}_σ of the mono-phase σ -model, iff the maps $\text{L}Q \rightarrow \text{L}M : \phi \mapsto \iota_\alpha \circ \phi$ are surjective submersions, the defect is topological and extra technical conditions are satisfied, cf Ref. [Sus11]. This is the case, in particular, when the defect carries the data of a 1-isomorphism $\Phi_F : \text{pr}_1^* \mathcal{G} \xrightarrow{\cong} \text{pr}_2^* \mathcal{G}$ over the graph $Q = (\text{id}_M \times F)(M)$ of an isometric diffeomorphism F of (M, g) . The associated duality relates mappings into a single component of M , and so it

represents a global symmetry of the mono-phase σ -model. This brings us to the main point of our discussion.

4. THE GAUGE PRINCIPLE THROUGH DEFECT NETWORKS

Global symmetries of the σ -model form a subgroup

$$G_\sigma \subset \text{Diff}(\mathcal{F})$$

acting on \mathcal{F} as

$$\ell : G_\sigma \times \mathcal{F} \rightarrow \mathcal{F},$$

with the ι_α and $\pi_n^{k,k+1}$ assumed G_σ -equivariant. Elements of G_σ satisfy the defining relation

$$S_\sigma[(\ell_g(X)|\Gamma); \eta] = S_\sigma[(X|\Gamma); \eta].$$

In order to understand the underlying geometry, it is instructive to examine infinitesimal symmetry transformations, induced by flows $\psi :]-\epsilon, \epsilon[\times \mathcal{F} \rightarrow \mathcal{F}$, $\epsilon > 0$ of the G_σ -fundamental vector fields $\mathcal{K} \in \mathfrak{X}(\mathcal{F})$. Inspection of the identity

$$\left. \frac{d}{dt} \right|_{t=0} S_\sigma[(\psi_t \circ X|\Gamma); \eta] = 0$$

reveals that the symmetries correspond to those sections

$$\mathcal{K} \oplus K \in \mathfrak{X}(\mathcal{F}) \oplus C_{\text{dR}}^1(M, Q, T_n | \Delta_Q, \Delta_{T_n})$$

of the generalised tangent bundles (here, $\Gamma((T_n \times \mathbb{R})_0) = \mathbb{R}^{\pi_0(T_n)}$)

$$\text{E}\mathcal{F} := (TM \oplus T^*M) \sqcup (TQ \oplus (Q \times \mathbb{R})) \sqcup \bigsqcup_{n \geq 3} (TT_n \oplus (T_n \times \mathbb{R})_0) \rightarrow \mathcal{F}$$

which obey the relations

$$\mathcal{L}_{\mathcal{K}} g = 0 \quad \text{and} \quad \tilde{\iota}_{\mathcal{K}} \Theta = -\tilde{d}K,$$

expressed in terms of the relative contraction

$$\tilde{\iota}_{\mathcal{V}}(\omega_M \oplus \omega_Q \oplus \omega_{T_n}) := \iota_{\mathcal{V}}\omega_M \oplus (-\iota_{\mathcal{V}}\omega_Q) \oplus \iota_{\mathcal{V}}\omega_{T_n},$$

cf Ref. [Sus12]. There is an essentially unique extension of the Lie bracket on $\mathfrak{X}(\mathcal{F})$ to $\Gamma(\text{E}\mathcal{F})$ that closes on the set $\Gamma_\sigma(\text{E}\mathcal{F})$ of these sections, to wit,

$$[\mathcal{V} \oplus v, \mathcal{W} \oplus w]_C^\Theta := [\mathcal{V}, \mathcal{W}] \oplus [\mathcal{L}_{\mathcal{V}} w - \mathcal{L}_{\mathcal{W}} v - \frac{1}{2} \tilde{d}(\tilde{\iota}_{\mathcal{V}} w - \tilde{\iota}_{\mathcal{W}} v) + \tilde{\iota}_{\mathcal{V}} \tilde{\iota}_{\mathcal{W}} \Theta],$$

and it is natural to think of it as a relative Θ -twisted Courant bracket. Indeed, it restricts to the H-twisted Courant bracket on sections of the standard generalised tangent bundle $TM \oplus T^*M$ whose rôle in the description of symmetries of the mono-phase σ -model has been known since Ref. [AS05]. The intrinsically gerbe-theoretic nature of the bracket is readily established with the help of Hitchin-type isomorphisms of Ref. [Sus12]. With the algebraic structure on $\Gamma_\sigma(\text{E}\mathcal{F})$ thus determined, we are ready to discuss the Gauge Principle.

Geometrically, the Principle stipulates that the covariant configuration bundle $\mathcal{F}' := \Sigma \times \mathcal{F}$ of the σ -model be replaced by the bundle $P \times_\ell \mathcal{F}$ associated, through ℓ , to an arbitrary principal G_σ -bundle $P \rightarrow \Sigma$ and carrying a fibrewise action of the adjoint bundle $P \times_{\text{Ad}} G_\sigma$. In this manner, the action of the symmetry group (encoded in the isotype G_σ of the fibre of $P \times_{\text{Ad}} G_\sigma$) is rendered local while preserving the original field content, the latter being determined by the isotype \mathcal{F} of the fibre of $P \times_\ell \mathcal{F}$. Clearly, we must insist that the new covariant configuration bundle admit a *global* section, to be identified with the lagrangean field of the gauged

σ -model under construction. The geometric descent from the extended covariant configuration bundle $\tilde{\mathcal{F}} := \mathbf{P} \times \mathcal{F}$ to $\mathbf{P} \times_{\ell} \mathcal{F}$ is straightforward as the G_{σ} -action on the former is free. The nontrivial task is to lift this action equivariantly to an extension $\tilde{\mathfrak{B}}_{\mathcal{A}}$ of the original \mathfrak{B} to $\tilde{\mathcal{F}}$ obtained by coupling its pullback to the gauge field given by the principal G_{σ} -connection 1-form

$$\mathcal{A} \in \Omega^1(\mathbf{P}) \otimes \mathfrak{g}_{\sigma}, \quad \mathfrak{g}_{\sigma} := \text{Lie } G_{\sigma}$$

on \mathbf{P} . This is to be done in such a manner that $\tilde{\mathfrak{B}}_{\mathcal{A}}$ descends to the quotient $\mathbf{P} \times_{\ell} \mathcal{F}$ in the sense captured by a variant, established in Refs. [GSW11, GSW10, GSW12], of Stevenson's Principle of Descent of Refs. [Ste00, Wal07] (to be viewed as a (2-)categorification of pullback) and defines a CFT invariant under the action of the gauge group $\mathbf{P} \times_{\text{Ad}} G_{\sigma}$.

It is convenient to begin the analysis with the topologically trivial case $\mathbf{P} = \Sigma \times G_{\sigma}$, in which we keep \mathcal{F}' as the covariant configuration bundle and replace \mathcal{A} by the connection 1-form $A = A^A \otimes t_A \in \Omega^1(\Sigma) \otimes \mathfrak{g}_{\sigma}$, written in terms of the generators t_A of \mathfrak{g}_{σ} subject to the relations

$$[t_A, t_B] = f_{ABC} t_C$$

in which the f_{ABC} are the structure constants of \mathfrak{g}_{σ} . Upon choosing a basis

$$\mathfrak{K}_A := \mathcal{K}_A \oplus (\kappa_A \oplus k_A \oplus 0), \quad A \in \overline{1, \dim \mathfrak{g}_{\sigma}}$$

in $\Gamma_{\sigma}(\mathbf{E}\mathcal{F})$ projecting to the set of G_{σ} -fundamental vector fields \mathcal{K}_A associated with the t_A , we obtain the following (minimal) extension $\mathfrak{B}'_{\mathcal{A}}$ of \mathfrak{B} to \mathcal{F}' (subscripts 1,2 denote pullbacks along the canonical projections) given in Refs. [GSW11, GSW10, GSW12]: the metric

$$g_A := g_2 - g(\mathcal{K}_A, \cdot)_2 \otimes A_1^A - A_1^A \otimes g(\mathcal{K}_A, \cdot)_2 + g(\mathcal{K}_A, \mathcal{K}_B)_2 A_1^A \otimes A_1^B$$

and the 0-cell

$$\mathcal{G}_A := \mathcal{G}_2 \otimes I_{\rho_A} \quad \text{with} \quad \rho_A := \kappa_{A2} \wedge A_1^A - \frac{1}{2} (\iota_{\mathcal{K}_A} \kappa_B)_2 A_1^A \wedge A_2^B$$

over M' ; the 1-cell

$$\Phi_A := \Phi_2 \otimes J_{\lambda_A}$$

with the trivial 1-cell (trivial line bundle) J_{λ_A} with a connection 1-form

$$\lambda_A := -k_{A2} A_1^A$$

over Q' ; the 2-cells

$$\varphi_{nA} := \varphi_{n2}$$

over the T'_n . In terms of these, we define – for $\xi := (\text{id}_{\Sigma}, X)$ with X as before – the gauged σ -model coupled to the topologically trivial gauge field

$$S_{\sigma}[(X|\Gamma); A, \gamma] := -\frac{1}{2} \int_{\Sigma} g_A(d\xi^{\wedge} \star_{\gamma} d\xi) - i \log \text{Hol}_{(\mathcal{G}_A, \Phi_A, \varphi_{nA} | n \in \mathbb{N}_{\geq 3})}(\xi|\Gamma)$$

We may next impose the demand that the above be invariant under gauge transformations

$$(X, A) \mapsto (\ell_X(X), \text{Ad}_X A - d_X \chi \chi^{-1})$$

defined for arbitrary $\chi \in C^\infty(\Sigma, G_\sigma)$. For χ homotopic to the identity, this yields the condition that the triple

$$\left(\bigoplus_{A=1}^{\dim \mathfrak{g}_\sigma} C^\infty(\mathcal{F}, \mathbb{R}) \mathcal{R}_A, [\cdot, \cdot]_C^\Theta, \alpha_{\mathcal{T}\mathcal{F}} \equiv \text{pr}_1 \right)$$

define a Lie algebroid, and the latter is then canonically isomorphic with the action algebroid $\mathfrak{g}_{\sigma \ltimes_\ell \mathcal{F}}$, cf Refs. [GSW11, GSW10, GSW12]. Invariance under large gauge transformations requires the existence of a 1-cell

$$\Upsilon : \ell^* \mathcal{G} \xrightarrow{\cong} \text{pr}_2^* \mathcal{G} \otimes I_{\rho_{\theta_L}}$$

(for θ_L the left-invariant Maurer–Cartan 1-form on G_σ) and a 2-cell

$$\Xi : \ell^* \Phi \xrightarrow{\cong} [((\text{id}_{G_\sigma} \times \iota_2)^* \Upsilon^{-1} \otimes \text{Id}) \circ (\text{pr}_2^* \Phi \otimes \text{Id}) \circ (\text{id}_{G_\sigma} \times \iota_1)^* \Upsilon] \otimes J_{\lambda_{\theta_L}}$$

(subject to additional constraints) from the 2-category $\mathfrak{B}\mathfrak{O}\mathfrak{r}\mathfrak{b}^\nabla(G_\sigma \times \mathcal{F})$ based on the arrow manifold of the action groupoid $G_{\sigma \ltimes_\ell \mathcal{F}}$. Upon recalling that $\mathfrak{g}_{\sigma \ltimes_\ell \mathcal{F}}$ is the tangent algebroid of $G_{\sigma \ltimes_\ell \mathcal{F}}$, we are led to identify the latter as the algebroid-differential structure underlying those symmetries of the σ -model that can be gauged. This observation was elucidated in [Sus12] with the help of the equivalence

$$G_\sigma\text{-}\mathfrak{B}\mathfrak{u}\mathfrak{n}(\Sigma \parallel \mathcal{F}) \cong G_{\sigma \ltimes_\ell \mathcal{F}}\text{-}\mathfrak{B}\mathfrak{u}\mathfrak{n}(\Sigma)$$

between the groupoid of principal G_σ -bundles $P \rightarrow \Sigma$ with the property that $P \times_\ell \mathcal{F}$ admits a global section and the groupoid of principal $G_{\sigma \ltimes_\ell \mathcal{F}}$ -bundles over Σ , the latter being defined in Ref. [Moe91]. The equivalence has a natural world-sheet interpretation, reminiscent of the categorial generalised-orbifold construction of Ref. [FFRS09]: An object P of $G_\sigma\text{-}\mathfrak{B}\mathfrak{u}\mathfrak{n}(\Sigma \parallel \mathcal{F})$ (data of the gauged σ -model, forgetting the connection) trivialising over an open cover $\mathcal{O} := \{\Sigma_i\}_{i \in \mathcal{I}}$ of Σ corresponds to a family of smooth mappings $X_i : \Sigma_i \rightarrow \mathcal{F}$ under which images of points $\sigma \in \Sigma_i \cap \Sigma_j$ are related as $X_i(\sigma) = \ell_{g_{ij}}(X_j)(\sigma)$ by arrows of $G_{\sigma \ltimes_\ell \mathcal{F}}$ defined in terms of transition maps g_{ij} of P . This demonstrates the necessity of incorporating nontrivial principal G_σ -bundles in a complete description of the gauged σ -model as they are necessary to reproduce the twisted sector of the descended CFT. Their presence appears to ensure self-consistency of the quantised theory, cf Refs. [GSW11, GSW10, GSW12].

The goal of extending the Gauge Principle to topologically nontrivial bundles P can be attained through local trivialisation of P over \mathcal{O} followed by the gluing of local data (X_i, A_i) over the Σ_i and of the local phases \mathfrak{B}'_{A_i} of the gauged σ -model by means of the g_{ij} at the edges $\ell_{ij} \subset \Sigma_i \cap \Sigma_j$ of a trivalent graph $\Gamma_{\mathcal{O}}$ associated with \mathcal{O} , with $\Gamma_{\mathcal{O}} \cap \Gamma$ discrete and not containing vertices of Γ . We then check, as in Ref. [Sus12], that the pair (Υ, Ξ) , required to exist by previous arguments, can be used to induce the desired structure of a topological defect at the ℓ_{ij} , alongside that of a (trans-)defect junction at $\Gamma_{\mathcal{O}} \cap \Gamma$. These jointly implement the gluing of local data in the presence of Γ , and the gluing itself emerges as a duality between the trivialised phases \mathfrak{B}'_{A_i} . The hitherto assignments already fix, by the arguments from Ref. [RS09], the structure to be pulled back to the vertices of $\Gamma_{\mathcal{O}}$, and so we are led to demand the existence of a 2-cell

$$\gamma : (d_2^{(0)*} \Upsilon \otimes \text{Id}) \circ d_2^{(2)*} \Upsilon \xrightarrow{\cong} d_2^{(1)*} \Upsilon$$

of $\overline{\mathfrak{B}\mathfrak{G}\mathfrak{r}\mathfrak{b}}^\nabla(G_\sigma^2 \times M)$, written in terms of the face maps $d_l^{(k)} : G_\sigma^l \times M \rightarrow G_\sigma^{l-1} \times M$, $k \in \overline{0, l}$ of the nerve of $G_\sigma \ltimes_\ell \mathcal{F}$ and playing a rôle analogous to that of φ_3 . At this stage, we still have to demand that the gauged σ -model thus sewn from the local phases \mathfrak{B}'_{A_i} be independent of the arbitrary choices made, *e.g.*, the choice of \mathcal{O} and $\Gamma_{\mathcal{O}}$, of the trivialisation and of index assignments. This further constrains (Y, γ, Ξ) , so that ultimately we have to demand that the triple define a G_σ -equivariant structure on \mathfrak{B} , as was demonstrated in Ref. [Sus12]. By the Principle of Descent, the gauged σ -model then yields a (manifestly gauge invariant) CFT on $P \times_\ell \mathcal{F}$, equivalent to a σ -model on \mathcal{F}/G_σ whenever the latter exists as a manifold, *cf* Refs. [GSW11, GSW10, GSW12]. Obstructions to the existence of the G_σ -equivariant structure (known as gauge anomalies), as well as its inequivalent realisations are classified in Refs. [GSW11, GSW10, GSW12] by a G_σ -equivariant extension of the relative Deligne hypercohomology.

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REFERENCES

- [Alv85] O. Alvarez, “*Topological quantization and cohomology*”, Commun. Math. Phys. **100** (1985), 279–309.
- [AS05] A.Yu. Alekseev and T. Strobl, “*Current algebra and differential geometry*”, JHEP **0503** (2005), 035.
- [FFRS04] J. Fröhlich, J.A. Fuchs, I. Runkel, and C. Schweigert, “*Kramers–Wannier duality from conformal defects*”, Phys. Rev. Lett. **93** (2004), 070601.
- [FFRS07] ———, “*Duality and defects in rational conformal field theory*”, Nucl. Phys. **B763** (2007), 354–430.
- [FFRS09] ———, “*Defect lines, dualities, and generalised orbifolds*”, Proceedings of the XVIth International Congress on Mathematical Physics: Prague, Czech Republic, 3–8 August 2009 (P. Exner, ed.), World Scientific, 2009, pp. 608–613.
- [FSW08] J.A. Fuchs, C. Schweigert, and K. Waldorf, “*Bi-branes: Target space geometry for world sheet topological defects*”, J. Geom. Phys. **58** (2008), 576–598.
- [Gaw88] K. Gawędzki, “*Topological Actions in Two-Dimensional Quantum Field Theory*”, Non-perturbative Quantum Field Theory (G. ’t Hooft, A. Jaffe, G. Mack, P. Mitter, and R. Stora, eds.), Plenum Press, 1988, pp. 101–141.
- [Gaw05] ———, “*Abelian and non-Abelian branes in WZW models and gerbes*”, Commun. Math. Phys. **258** (2005), 23–73.
- [GR02] K. Gawędzki and N. Reis, “*WZW branes and gerbes*”, Rev. Math. Phys. **14** (2002), 1281–1334.
- [GR03] ———, “*Basic gerbe over non simply connected compact groups*”, J. Geom. Phys. **50** (2003), 28–55.
- [GSW10] K. Gawędzki, R. R. Suszek, and K. Waldorf, “*Global gauge anomalies in two-dimensional bosonic sigma models*”, Commun. Math. Phys. **302** (2010), 513–580.
- [GSW11] ———, “*Bundle gerbes for orientifold sigma models*”, Adv. Theor. Math. Phys. **15** (2011), 621–688.
- [GSW12] ———, “*The gauging of two-dimensional bosonic sigma models on world-sheets with defects*”, arXiv preprint: 1202.5808 [hep-th], submitted to Rev. Math. Phys.
- [Moe91] I. Moerdijk, “*Classifying toposes and foliations*”, Ann. Inst. Fourier **41** (1991), 189–209.
- [MS00] M.K. Murray and D. Stevenson, “*Bundle gerbes: stable isomorphism and local theory*”, J. Lond. Math. Soc. **62** (2000), 925–937.
- [Mur96] M.K. Murray, “*Bundle gerbes*”, J. Lond. Math. Soc. **54** (1996), 403–416.
- [RS09] I. Runkel and R. R. Suszek, “*Gerbe-holonomy for surfaces with defect networks*”, Adv. Theor. Math. Phys. **13** (2009), 1137–1219.

- [Ste00] D. Stevenson, “*The Geometry of Bundle Gerbes*”, Ph.D. thesis, The University of Adelaide, Australia, 2000.
- [Sus11] R. R. Suszek, “*Defects, dualities and the geometry of strings via gerbes. I. Dualities and state fusion through defects*”, Hamb. Beitr. Math. Nr. **360** (2011) [arXiv preprint: 1101.1126 [hep-th]].
- [Sus12] ———, “*Defects, dualities and the geometry of strings via gerbes II. Generalised geometries with a twist, the gauge anomaly and the gauge-symmetry defect*”, Hamb. Beitr. Math. Nr. **361** (2011) [arXiv preprint: 1209.2334 [hep-th]].
- [Wal07] K. Waldorf, “*More morphisms between bundle gerbes*”, Theory Appl. Categories **18** (2007), 240–273.

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